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THE DIRECTIONAL STABILITY OF TOWED SHIPS WITH PARTICULAR REFERENCE TO BARGES

BRUEN HENRY PETERS

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THE DIRECTIONAL STABILITY OF TOWED SHIPS WITH PARTICULAR REFERENCE TO BARGES

A Thesis Submitted in Partial Fulfillment
of the Requirements for the Degree of
Master of Science

STEVENS INSTITUTE OF TECHNOLOGY

Submitted by Bruen H. Peters

Thesis Adviser
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TABLE OF CONTENTS

Page

Introduction 1

Mathematical Development 2

    Kinematic Relations 3
    Differential Equations of Motion 3
    Linear Simplifications 6
    Non-Dimensionalization of Equations of Motion .. 9

Stability Analysis 12

Simplified Theory 21

Numerical Analysis 30

Conclusions 34

Appendix I: Nomenclature 36

References 38

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INTRODUCTION

In an academic sense alone, a mathematical investigation of the stability of towed bodies and systems is of interest only if it adds to the existing knowledge in the general field of the course-keeping qualities of vessels. In a more practical sense, it is an important and, in fact, necessary primary design aid which yields specific criteria to be satisfied for stable operation. These criteria are composed of terms which, for the most part, have distinct physical significance and which can be varied in design studies. Through an appropriate analysis of such terms, desirable operating characteristics may be found to a considerable degree of accuracy while such a design is still in the "drafting board" stage.

This paper deals with the analysis of a system composed of a towboat or tug and a single barge. Both bodies are considered to be unrestrained, hence the stability of the system will be dependent on the characteristics of the component parts. A simple automatic control is specified for the towing body in order that the system should have directional rather than dynamic stability (or instability).

In general the methods followed are those of Euler, Routh and Hurwitz and deal with systems of simple linear differential equations only. Laplace Transformations were not applied, since the author has found that many engineers are unfamiliar with the properties of this concept.

MATHEMATICAL DEVELOPMENT

The subsequent analysis presupposes the following basic simplifying assumptions:

1. Both bodies have freedom in the horizontal plane alone, i.e., roll and pitch are ignored.

2. The towline is assumed horizontal and taut at all times.

3. Angular displacements are considered to be small, so that second order and higher terms may be neglected in the final equations of motion.

With these premises in mind, consider a set of right hand orthogonal axes fixed relative to the earth and denoted by (x_0, y_0). Consider also similar systems of axes (x, y) and $(\bar{x}, \bar{y})$ fixed in the barge and towboat respectively. Such a system is shown in Fig. 1 together with other symbols necessary to define completely the orientation of the system at any time. A complete and detailed nomenclature is given in Appendix I.

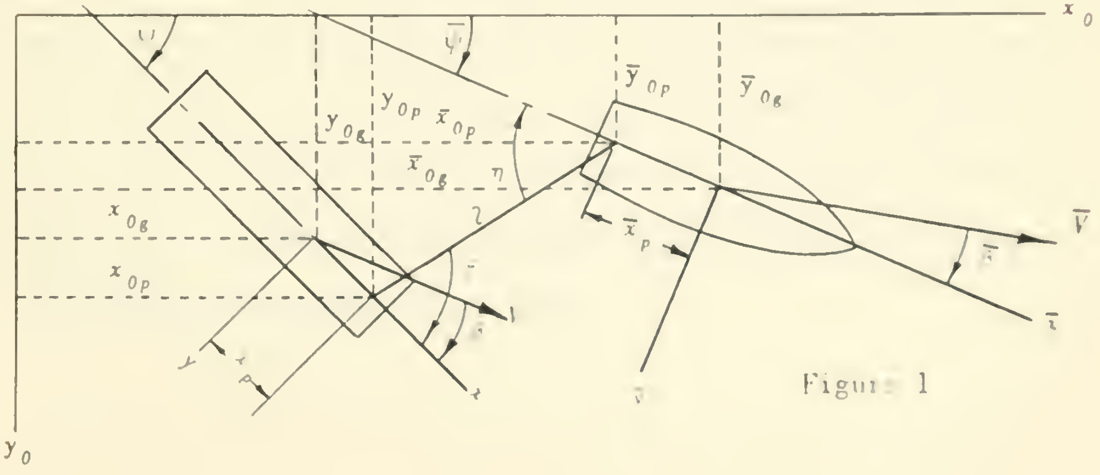


Figure 1

Kinematic Relations

Certain expressions are obtained from the towing geometry. These are presented without comment based on Fig. 1 of Ref. 1.

$$
\begin{aligned}
y_{0p} &= \int V \,\mathrm{Sin}\,(\psi - \beta)dt + x_p \,\mathrm{Sin}\psi = \bar{y}_{0p} + l \,\mathrm{Sin}\,(\eta - \bar{\psi}) \\
x_{0p} &= \int V \,\mathrm{Cos}\,(\psi - \beta)dt + x_p \,\mathrm{Cos}\psi = \bar{x}_{0p} - l \,\mathrm{Cos}\,(\eta - \bar{\psi}) \\
\bar{y}_{0p} &= \int \bar{V} \,\mathrm{Sin}\,(\bar{\psi} - \bar{\beta})dt + \bar{x}_p \,\mathrm{Sin}\bar{\psi} \\
\bar{x}_{0p} &= \int \bar{V} \,\mathrm{Cos}\,(\bar{\psi} - \bar{\beta})dt + \bar{x}_p \,\mathrm{Cos}\bar{\psi}
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
\therefore \eta &= \bar{\psi} + \mathrm{arc\ Cos}\left[\int \frac{\bar{V}}{l} \,\mathrm{Cos}\,(\bar{\psi} - \bar{\beta})dt - \int \frac{V}{l} \,\mathrm{Cos}\,(\psi - \beta)dt + \frac{\bar{x}_p}{l} \,\mathrm{Cos}\bar{\psi} - \frac{x_p}{l} \,\mathrm{Cos}\psi\right] \\
&= \bar{\psi} + \mathrm{arc\ Sin}\left[\int \frac{V}{l} \,\mathrm{Sin}\,(\psi - \beta)dt - \int \frac{\bar{V}}{l} \,\mathrm{Sin}\,(\bar{\psi} - \bar{\beta})dt + \frac{x_p}{l} \,\mathrm{Sin}\psi - \frac{\bar{x}_p}{l} \,\mathrm{Sin}\bar{\psi}\right]
\end{aligned}
$$

$$
\xi - \psi = \eta - \bar{\psi} \tag{2}
$$

$$
\begin{aligned}
T_x &= T \,\mathrm{Cos}\xi \quad , \quad T_y = - T \,\mathrm{Sin}\xi \quad , \quad N_{(T)} = - x_p T \,\mathrm{Sin}\xi \\
\bar{T}_x &= - T \,\mathrm{Cos}\eta \quad , \quad \bar{T}_y = T \,\mathrm{Sin}\eta \quad , \quad \bar{N}_{(T)} = \bar{x}_p \, T \,\mathrm{Sin}\eta
\end{aligned} \tag{3}
$$

Differential Equations of Motion

Since axes fixed in the bodies are essential to the development it is necessary to follow Euler, Ref. 2 in determining appropriate mass-acceleration terms. For the purpose of continuity these expressions will now be derived.

Choose a set of axes fixed in space and a set of axes fixed in the body as shown in Fig. 1. If P_x and P_y are the momentum expressions relative to the axes fixed in the body then (according to Newton) the force components along the axes fixed in space are equal to the components of the time rate of change of momentum along these axes. Thus

$$X_0 = \frac{d}{dt} P_{0x} = \frac{d}{dt}\left[P_x \operatorname{Cos}\psi - P_y \operatorname{Sin}\psi \right]$$

$$Y_0 = \frac{d}{dt} P_{0y} = \frac{d}{dt}\left[P_x \operatorname{Sin}\psi + P_y \operatorname{Cos}\psi \right] \tag{4}$$

also

$$X_0 = X \operatorname{Cos}\psi - Y \operatorname{Sin}\psi \qquad Y_0 = X \operatorname{Sin}\psi + Y \operatorname{Cos}\psi \tag{5}$$

$$\therefore X = \dot{P}_x - P_y \dot{\psi} \qquad Y = \dot{P}_y + P_x \dot{\psi} \tag{6}$$

where

$$P_x = M_1 u \qquad P_y = M_2 v \quad . \tag{7}$$

Then

$$X = M_1 \dot{u} - M_2 v \dot{\psi} \qquad Y = M_2 \dot{v} + M_1 u \dot{\psi} \tag{8}$$

Similarly

$$\bar{X} = \bar{M}_1 \dot{u} - M_2 v \dot{\bar{\psi}} \qquad \bar{Y} = \bar{M}_2 \dot{\bar{v}} + \bar{M}_1 \bar{u} \dot{\bar{\psi}} \quad . \tag{9}$$

The moment of inertia terms are

$$N = I \ddot{\psi} \quad \text{and} \quad \bar{N} = \bar{I} \ddot{\bar{\psi}} \tag{10}$$

We consider here Ref. 1 and Ref. 2 and note that

$$M_1 = M(1 + k_1) \quad , \quad M_2 = M(1 + k_2) \qquad I = I_B + I_w k' \tag{11}$$

where M is the absolute mass of the ship; I_B is the moment of inertia of the body about a vertical axis taken through its center of gravity, I_w is the moment of inertia of the fluid displaced by the body taken about a vertical axis through its center of gravity and k_1, k_2 and k' are appropriate coeffi-

cients of accession to inertia. Similar expressions may be written for the "bar" terms, thus

$$\bar{M}_1 = \bar{M}\,(1 + \bar{k}_1) \qquad \bar{M}_2 = \bar{M}\,(1 + \bar{k}_2)$$
$$\bar{I} = \bar{I}_B + \bar{I}_w\,\bar{k}' \tag{12}$$

We are now in a position to write the differential equations of motion:

1. Barge

   a) Longitudinal Force

   $$M_1\,\dot{u} - M_2\,v\,\dot{\psi} = X\,(u, v, \dot{\psi}) + T_x \tag{13}$$

   b) Lateral Force

   $$M_2\,\dot{v} + M_1\,u\,\dot{\psi} = Y\,(u, v, \dot{\psi}) + T_y \tag{14}$$

   c) Yawing Moment

   $$I\,\ddot{\psi} = N\,(u, v, \dot{\psi}) + N_{(T)} \tag{15}$$

2. Towboat

   a) Longitudinal Force

   $$\bar{M}_1\,\dot{\bar{u}} - \bar{M}_2\,\bar{v}\,\dot{\bar{\psi}} = \bar{X}\,(\bar{u}, \bar{v}, \dot{\bar{\psi}}, \bar{\delta}) + \bar{T}_x \tag{16}$$

   b) Lateral Force

   $$\bar{M}_2\,\dot{\bar{v}} + \bar{M}_1\,\bar{u}\,\dot{\bar{\psi}} = \bar{Y}\,(\bar{u}, \bar{v}, \dot{\bar{\psi}}, \bar{\delta}) + \bar{T}_y \tag{17}$$

   c) Yawing Moment

   $$\bar{I}\,\ddot{\bar{\psi}} = \bar{N}\,(\bar{u}, \bar{v}, \dot{\bar{\psi}}, \bar{\delta}) + \bar{N}_{(T)} \tag{18}$$

3. The towboat rudder equations for rudder angle proportional to heading change are:

   a) Rudder order

   $$\bar{\delta}^{\bullet} = a_0\,\bar{\omega} \tag{19}$$

   b) Constant time lag

   $$\bar{\delta}^{\bullet} = \bar{\delta} + \bar{t}\,\frac{d\bar{\delta}}{dt} \tag{20}$$

Linear Simplifications

Unless the previous systems are linearized, a classical solution is extremely difficult to obtain, in fact in a nonlinear form the problem can best be treated through the use of automatic computing devices. However, only a closed form solution will be considered herein, hence the systems will now be linearized.

It is first assumed that $V \approx \bar{V}$, i.e. that both the towboat and barge are traveling at the same speed and that both β and $\bar{\beta}$ take on only small values. Then the following relations are obtained from Figure 1

$$\begin{aligned} u &= V \operatorname{Cos}\beta \approx V \\ v &= -V \operatorname{Sin}\beta \approx -V\beta \\ \bar{u} &= V \operatorname{Cos}\bar{\beta} \approx V \\ \bar{v} &= -V \operatorname{Sin}\bar{\beta} \approx -V\bar{\beta} \end{aligned} \tag{21}$$

Noting these relations, the truth of the following identities is obvious

$$\begin{aligned} X\ (u, v, \dot{\psi}) &= X\ (\beta, \dot{\psi}) \\ Y\ (u, v, \dot{\psi}) &= Y\ (\beta, \dot{\psi}) \\ N\ (u, v, \dot{\psi}) &= N\ (\beta, \dot{\psi}) \\ \bar{X}\ (\bar{u}, \bar{v}, \dot{\bar{\psi}}, \bar{\delta}) &= \bar{X}\ (\bar{\beta}, \dot{\bar{\psi}}, \bar{\delta}) \\ \bar{Y}\ (\bar{u}, \bar{v}, \dot{\bar{\psi}}, \bar{\delta}) &= \bar{Y}\ (\bar{\beta}, \dot{\bar{\psi}}, \bar{\delta}) \\ \bar{N}\ (\bar{u}, \bar{v}, \dot{\bar{\psi}}, \bar{\delta}) &= \bar{N}\ (\bar{\beta}, \dot{\bar{\psi}}, \bar{\delta}) \end{aligned} \tag{22}$$

Limiting all displacements to the first order of small quantities, relations (22) may be expanded in Taylor's Series to obtain the following:

$$
\begin{aligned}
X(\beta,\dot{\psi}) &= X_e + \frac{\partial X}{\partial \beta}(\beta - \beta_e) + \frac{\partial X}{\partial \dot{\psi}}(\dot{\psi} - \dot{\psi}_e) \\
Y(\beta,\dot{\psi}) &= Y_e + \frac{\partial Y}{\partial \beta}(\beta - \beta_e) + \frac{\partial Y}{\partial \dot{\psi}}(\dot{\psi} - \dot{\psi}_e) \\
N(\beta,\dot{\psi}) &= N_e + \frac{\partial N}{\partial \beta}(\beta - \beta_e) + \frac{\partial N}{\partial \dot{\psi}}(\dot{\psi} - \dot{\psi}_e) \qquad (23) \\
\bar{X}(\bar{\beta},\dot{\bar{\psi}},\bar{\delta}) &= \bar{X}_e + \frac{\partial \bar{X}}{\partial \bar{\beta}}(\bar{\beta} - \bar{\beta}_e) + \frac{\partial \bar{X}}{\partial \dot{\bar{\psi}}}(\dot{\bar{\psi}} - \dot{\bar{\psi}}_e) + \frac{\partial \bar{X}}{\partial \bar{\delta}}(\bar{\delta} - \bar{\delta}_e) \\
\bar{Y}(\bar{\beta},\dot{\bar{\psi}},\bar{\delta}) &= \bar{Y}_e + \frac{\partial \bar{Y}}{\partial \bar{\beta}}(\bar{\beta} - \bar{\beta}_e) + \frac{\partial \bar{Y}}{\partial \dot{\bar{\psi}}}(\dot{\bar{\psi}} - \dot{\bar{\psi}}_e) + \frac{\partial \bar{Y}}{\partial \bar{\delta}}(\bar{\delta} - \bar{\delta}_e) \\
\bar{N}(\bar{\beta},\dot{\bar{\psi}},\bar{\delta}) &= \bar{N}_e + \frac{\partial \bar{N}}{\partial \bar{\beta}}(\bar{\beta} - \bar{\beta}_e) + \frac{\partial \bar{N}}{\partial \dot{\bar{\psi}}}(\dot{\bar{\psi}} - \dot{\bar{\psi}}_e) + \frac{\partial \bar{N}}{\partial \bar{\delta}}(\bar{\delta} - \bar{\delta}_e)
\end{aligned}
$$

where the subscript e denotes the equilibrium values about which the expansion takes place. Since it is assumed that initially the system is on straight course and in line it is obvious that:

$$Y_e = N_e = \bar{Y}_e = \bar{N}_e = 0$$

and

$$\beta_e = \dot{\psi}_e = \bar{\beta}_e = \dot{\bar{\psi}}_e = \bar{\delta}_e = 0 \quad .$$

In addition, for reasons of symmetry, Ref. 3,

$$\frac{\partial X}{\partial \beta} = \frac{\partial \bar{X}}{\partial \bar{\beta}} = \frac{\partial X}{\partial \dot{\psi}} = \frac{\partial \bar{X}}{\partial \dot{\bar{\psi}}} = \frac{\partial \bar{X}}{\partial \bar{\delta}} = 0 \quad .$$

Relations (22) then reduce to:

$$
\begin{aligned}
X &= X_e \qquad Y = Y_\beta \beta + Y_{\dot{\psi}} \dot{\psi} \\
N &= N_\beta \beta + N_{\dot{\psi}} \dot{\psi} \qquad (24) \\
\bar{X} &= \bar{X}_e \qquad \bar{Y} = \bar{Y}_{\bar{\beta}} \bar{\beta} + \bar{Y}_{\dot{\bar{\psi}}} \dot{\bar{\psi}} + \bar{Y}_{\bar{\delta}} \bar{\delta} \\
\bar{N} &= \bar{N}_{\bar{\beta}} \bar{\beta} + \bar{N}_{\dot{\bar{\psi}}} \dot{\bar{\psi}} + \bar{N}_{\bar{\delta}} \bar{\delta}
\end{aligned}
$$

where the subscript notation has replaced the conventional partial derivative form.

Continuing the linearization to include expressions (1), (2), and (3) the following relations are established

$$
\begin{aligned}
T_x &= T \\
T_y &= -T\left[\psi + \int \frac{V}{l}(\psi-\beta)dt - \int \frac{V}{l}(\bar{\psi}-\bar{\beta})dt + \frac{x_p}{l}\psi - \frac{\bar{x}_p}{l}\bar{\psi}\right] \\
N_{(T)} &= -x_p T\left[\psi + \int \frac{V}{l}(\psi-\beta)dt - \int \frac{V}{l}(\bar{\psi}-\bar{\beta})dt + \frac{x_p}{l}\psi - \frac{\bar{x}_p}{l}\bar{\psi}\right] \\
\bar{T}_x &= -T \\
\bar{T}_y &= T\left[\bar{\psi} + \int \frac{V}{l}(\psi-\beta)dt - \int \frac{V}{l}(\bar{\psi}-\bar{\beta})dt + \frac{x_p}{l}\psi - \frac{\bar{x}_p}{l}\bar{\psi}\right] \\
\bar{N}_{(\bar{T})} &= \bar{x}_p T\left[\bar{\psi} + \int \frac{V}{l}(\psi-\beta)dt - \int \frac{V}{l}(\bar{\psi}-\bar{\beta})dt + \frac{x_p}{l}\psi - \frac{\bar{x}_p}{l}\bar{\psi}\right]
\end{aligned}
\tag{25}
$$

In a similar fashion expressions (8), (9) and (10) become

$$
\begin{aligned}
X &= 0 \\
Y &= M_1 V \dot{\psi} - M_2 V \dot{\beta} \\
N &= I \ddot{\psi} \\
\bar{X} &= 0 \\
\bar{Y} &= \bar{M}_1 V \dot{\bar{\psi}} - \bar{M}_2 V \dot{\bar{\beta}} \\
\bar{N} &= \bar{I} \ddot{\bar{\psi}}
\end{aligned}
\tag{26}
$$

With these linearizations the equations of motion become:

1. Barge

   a) Longitudinal Force

   $$0 = X_e + T \tag{27}$$

   b) Lateral Force

   $$
   \begin{aligned}
   M_1 V \dot{\psi} - M_2 V \dot{\beta} = Y_\beta \beta + Y_{\dot{\psi}} \dot{\psi} \\
   -T\left[\psi + \int \frac{V}{l}(\psi-\beta)dt - \int \frac{V}{l}(\bar{\psi}-\bar{\beta})dt\right] \\
   -T\left[\frac{x_p}{l}\psi - \frac{\bar{x}_p}{l}\bar{\psi}\right]
   \end{aligned}
   \tag{28}
   $$

c) Yawing Moment

$$I\ddot{\psi} = N_{\beta}\beta + N_{\dot{\psi}}\dot{\psi} - x_p T\left[\psi + \int\frac{V}{l}(\psi-\beta)dt - \int\frac{V}{l}(\bar{\psi}-\bar{\beta})dt\right] - x_p T\left[\frac{x}{l}p\,\psi - \frac{\bar{x}}{l}p\,\bar{\psi}\right] \qquad (29)$$

2. Towboat

a) Longitudinal Force

$$0 = \bar{Y}_e - T \qquad (30)$$

b) Lateral Force

$$\bar{M}_1 V\dot{\bar{\psi}} - \bar{M}_2 V\dot{\bar{\beta}} = \bar{Y}_{\bar{\beta}}\bar{\beta} + \bar{Y}_{\dot{\bar{\psi}}}\dot{\bar{\psi}} + \bar{Y}_{\bar{\delta}}\bar{\delta} + T\left[\bar{\psi} + \int\frac{V}{l}(\psi-\beta)\,dt\right] - T\left[\int\frac{V}{l}(\bar{\psi}-\bar{\beta})dt + \frac{\bar{x}}{l}p\,\bar{\psi} - \frac{x}{l}p\,\psi\right] \qquad (31)$$

c) Yawing Moment

$$\bar{I}\ddot{\bar{\psi}} = \bar{N}_{\bar{\beta}}\bar{\beta} + \bar{N}_{\dot{\bar{\psi}}}\dot{\bar{\psi}} + \bar{N}_{\bar{\delta}}\bar{\delta} + \bar{x}_p T\left[\bar{\psi} + \int\frac{V}{l}(\psi-\beta)dt - \int\frac{V}{l}(\bar{\psi}-\bar{\beta})dt\right] + \bar{x}_p T\left[\frac{x}{l}p\,\psi - \frac{\bar{x}}{l}p\,\bar{\psi}\right] \qquad (32)$$

d) Rudder Order

$$\bar{\delta}^{*} = a_o\bar{\psi} \qquad (33)$$

e) Constant Time Lag

$$\bar{\delta}^{*} = \bar{\delta} + \bar{t}\,\frac{d\bar{\delta}}{dt} \qquad (34)$$

Equations (27) and (30) represent steady-state conditions and need not be considered further in the stability analysis.

Non-Dimensionalization of Equations of Motion

Since the hydrodynamic forces may best be expressed in terms of dimensionless coefficients, one divides equation

(28) by $\frac{\rho}{2}\ell^2 V^2$; (29) by $\frac{\rho}{2}\ell^3 V^2$

(31) by $\frac{\rho}{2}\bar{\ell}^2 V^2$; and (32) by $\frac{\rho}{2}\bar{\ell}^3 V^2$

Furthermore the following transformation is introduced for the independent variable t $s = \frac{Vt}{L}$.

Here s is the number of tow lengths traveled, t the true time, L the tow length and V the linear velocity of the system. Since V is considered substantially constant the following relation is written $\frac{ds}{dt} = \frac{V}{L}$. (35)

thus the following expressions are obtained:

$$\dot{\psi} = \frac{d\psi}{ds}\ \frac{ds}{dt} = \dot{\psi}'\left[\frac{V}{L}\right] \qquad \ddot{\psi} = \frac{d\dot{\psi}}{ds}\ \frac{ds}{dt} = \ddot{\psi}'\left[\frac{V}{L}\right]^{2}$$

$$\dot{\beta} = \frac{d\beta}{ds}\ \frac{ds}{dt} = \dot{\beta}'\left[\frac{V}{L}\right]$$

$$\dot{\bar{\psi}} = \frac{d\bar{\psi}}{ds}\ \frac{ds}{dt} = \dot{\bar{\psi}}'\left[\frac{V}{L}\right], \qquad \ddot{\bar{\psi}} = \frac{d\dot{\bar{\psi}}}{ds}\ \frac{ds}{dt} = \ddot{\bar{\psi}}'\left[\frac{V}{L}\right]^{2}$$

$$\dot{\bar{\beta}} = \frac{d\bar{\beta}}{ds}\ \frac{ds}{dt} = \dot{\bar{\beta}}'\left[\frac{V}{L}\right]$$

In addition

$$m_1' = \frac{M_1}{\frac{\rho}{2}\ell^3}, \qquad m_2' = \frac{M_2}{\frac{\rho}{2}\ell^3}, \qquad n' = \frac{I}{\frac{\rho}{2}\ell^5}$$

$$\ell' = \frac{\ell}{L} \qquad \bar{\ell}' = \frac{\bar{\ell}}{L} \qquad \text{ʅ}' = \frac{\text{ʅ}}{L} \tag{36}$$

$$\bar{m}_1' = \frac{\bar{M}_1}{\frac{\rho}{2}\bar{\ell}^3} \qquad \bar{m}_2' = \frac{\bar{M}_2}{\frac{\rho}{2}\bar{\ell}^3} \qquad \bar{n}' = \frac{\bar{I}}{\frac{\rho}{2}\bar{\ell}^5}$$

Similarly:

$$Y'_\beta = \frac{Y_\beta}{\frac{\rho}{2}\ell^2V^2} \qquad N'_\beta = \frac{N_\beta}{\frac{\rho}{2}\ell^3\,V^2}, \qquad T' = \frac{T}{\frac{\rho}{2}\ell^2V^2}$$

$$Y'_{\dot{\psi}} = \frac{Y_{\dot{\psi}}}{\frac{\rho}{2}\ell^2V^2}\left[\frac{V}{L}\right], \qquad N'_{\dot{\psi}} = \frac{N_{\dot{\psi}}}{\frac{\rho}{2}\ell^3V^2}\left[\frac{V}{L}\right]$$

And:

$$\bar{Y}'_{\bar{\beta}} = \frac{\bar{Y}_{\bar{\beta}}}{\frac{\rho}{2}\bar{\ell}^2V^2} \qquad \bar{N}'_{\bar{\beta}} = \frac{\bar{N}_{\bar{\beta}}}{\frac{\rho}{2}\bar{\ell}^3V^2}$$

$$\bar{Y}'_{\dot{\bar{\psi}}} = \frac{\bar{Y}_{\dot{\bar{\psi}}}}{\frac{\rho}{2}\bar{\ell}^2V^2}\left[\frac{V}{L}\right], \qquad \bar{N}'_{\dot{\bar{\psi}}} = \frac{\bar{N}_{\dot{\bar{\psi}}}}{\frac{\rho}{2}\bar{\ell}^3V^2}\left[\frac{V}{L}\right]$$

Using the foregoing relations the equations of motion become:

1. Barge

a) Lateral Force

$$m_1' \ell' \ddot{\psi}' - m_2' \ell' \dot{\beta}' - Y'_{\beta}\beta - Y'_{\dot{\psi}}\dot{\psi}' \tag{37}$$

$$+ \frac{T'}{\zeta'}\left[\int \{(\psi - \bar{\psi}) + (\bar{\beta} - \beta)\}\, ds + (\ell' x_p' + \zeta')\psi - \bar{\ell}' \bar{x}_p' \bar{\psi}\right] = 0$$

b) Yawing Moment

$$n \ell'^{2} \ddot{\psi}' - N'_{\beta}\beta - N'_{\dot{\psi}}\dot{\psi}' \tag{38}$$

$$+ \frac{x_p' T'}{\zeta'}\left[\int \{(\psi - \bar{\psi}) + (\bar{\beta} - \beta)\}\, ds + (\ell' x_p' + \zeta')\psi - \ell' \bar{x}_p' \bar{\psi}\right]$$

2 Towboat

a) Lateral Force

$$\bar{m}_1' \bar{\ell}' \dot{\bar{\psi}}' - \bar{m}_2' \bar{\ell}' \dot{\bar{\beta}}' - \bar{Y}'_{\bar{\beta}}\bar{\beta} - \bar{Y}'_{\dot{\bar{\psi}}}\dot{\bar{\psi}}' - \bar{Y}'_{\bar{\delta}}\bar{\delta} \tag{39}$$

$$- \frac{T'}{\zeta'}\left[\int \{(\psi - \bar{\psi}) + (\bar{\beta} - \beta)\}\, ds + x_p' \psi + (\zeta' - \bar{\ell}' \bar{x}_p')\bar{\psi}\right] = 0$$

b) Yawing Moment

$$\bar{n}' \bar{\ell}'^{2} \dot{\bar{\psi}}' - \bar{N}_{\bar{\beta}}\bar{\beta} - \bar{N}_{\dot{\bar{\psi}}}\dot{\bar{\psi}} - \bar{N}_{\bar{\delta}}\bar{\delta} \tag{40}$$

$$- \frac{\bar{x}_p' T'}{\zeta'}\left[\int \{(\psi - \bar{\psi}) + (\bar{\beta} - \beta)\}\, ds + \ell' x_p' \psi + (\zeta' - \bar{\ell}' \bar{x}_p')\bar{\psi}\right]$$

c) Rudder Order

$$\bar{\delta}^{*} = a_o \bar{\psi} \tag{41}$$

d) Constant Time Lag

$$\bar{\delta}^{*} = \bar{\delta} + \bar{s}\,\frac{d\bar{\delta}}{ds} \tag{42}$$

STABILITY ANALYSIS

The equations of motion of the bodies under consideration together with the coupling effects due to the towline constraint and the imposition of the control functions on the towing body have been presented in the previous section.

These equations are of general significance within the limits of linear theory with the exception that a constant time lag has been assumed for the control function. This assumption has been made purely to show that time lags of a sort can be included in the development. However in the subsequent analysis the control time lag $\overline{s}$ will be assumed to be zero. No loss in generality will occur since it is evident that the inclusion of such a term will only serve to increase the order of the stability equation by one. Its exclusion will not make the analysis any less valid but will to some degree serve to simplify it.

To remove the integral signs in equations (37), (38), (39) and (40), we operate on them by $\frac{d}{ds}$ and hence raise their order by one. The equations then become:

1. Barge

   a) Lateral Force

$$\left[m_1'\ell' - Y'_{\dot{\psi}}\right]\ddot{\psi}' + \frac{T'}{l'}\left[\ell' x_p' + l'\right]\dot{\psi}' + \frac{T'}{l'}\psi - m_2'\ell'\ddot{\beta}' - Y'_{\beta}\dot{\beta}' - \frac{T'}{l'}\beta - \frac{T'}{l'}\bar{\ell}'\bar{x}_p'\dot{\bar{\psi}}' - \frac{T'}{l'}\bar{\psi} + \frac{T'}{l'}\bar{\beta} = 0 \qquad (43)$$

b) Yawing Moment

$$n'\ell'^2 \dddot{\psi}' - N'_{\dot{\psi}}\ddot{\psi}' + \frac{x_p' \; T'}{\zeta'} (\ell' x_p' + \zeta') \dot{\psi}' + \frac{x_p' \; T'}{\zeta'} \psi - N'_{\beta}\dot{\beta}' - \frac{x_p' \; T'}{\zeta'} \beta - \frac{x_p' \; \bar{x}_p' \; T'}{\zeta'} \dot{\bar{\psi}}' - \frac{x_p' \; T'}{\zeta'} \bar{\psi} + \frac{x_p' \; T'}{\zeta'} \bar{\beta} = 0 \qquad (44)$$

2. Towboat

a) Lateral Force

$$\left[\bar{m}_1'\bar{\ell}' - \bar{Y}'_{\dot{\bar{\psi}}}\right]\ddot{\bar{\psi}}' - \frac{T'}{\zeta'}\left[\zeta' - \bar{\ell}'\bar{x}_p'\right]\dot{\bar{\psi}}' + \frac{T'}{\zeta'}\bar{\psi} - \bar{m}_2'\bar{\ell}'\ddot{\bar{\beta}}' - \bar{Y}'_{\bar{\beta}}\dot{\bar{\beta}}' - \frac{T'}{\zeta'}\bar{\beta} - \frac{T'}{\zeta'}\ell' x_p' \; \dot{\psi}' - \frac{T'}{\zeta'}\psi + \frac{T'}{\zeta'}\beta - \bar{Y}'_{\bar{\delta}}\dot{\bar{\delta}}' = 0 \qquad (45)$$

b) Yawing Moment

$$\bar{n}'\bar{\ell}'^2 \dddot{\bar{\psi}}' - \bar{N}'_{\dot{\bar{\psi}}}\ddot{\bar{\psi}}' - \frac{\bar{x}_p' \; T'}{\zeta'} (\zeta' - \bar{\ell}'\bar{x}_p') \dot{\bar{\psi}}' + \frac{\bar{x}_p \; T'}{\zeta'}\bar{\psi} - \bar{N}'_{\bar{\beta}}\dot{\bar{\beta}}' - \frac{x_p' \; T'}{\zeta'} \bar{\beta} - \frac{\bar{x}_p' \; x_p' \ell' T'}{\zeta'} \dot{\psi}' - \frac{\bar{x}_p' \; T'}{\zeta'}\psi + \frac{\bar{x}_p' \; T'}{\zeta'}\beta - \bar{N}'_{\bar{\delta}}\dot{\bar{\delta}}' = 0 \qquad (46)$$

c) Rudder

$$\bar{\delta} = a_o \bar{\psi} \qquad (47)$$

These are linear homogeneous equations in the dependent variables ψ, β, $\bar{\psi}$, $\bar{\beta}$, $\bar{\delta}$, hence their solutions, excluding the highly improbable cases where the roots are equal, are given by the following equations:

$$\psi = \sum_i \psi_i \, e^{\sigma i s} \qquad \beta = \sum_i \beta_i \, e^{\sigma i s}$$
$$\bar{\psi} = \sum_i \bar{\psi}_i \, e^{\sigma i s} \qquad \bar{\beta} = \sum_i \bar{\beta}_i \, e^{\sigma i s} \qquad (48)$$
$$\bar{\delta} = \sum_i \bar{\delta}_i \, e^{\sigma i s}$$

It is our aim to determine:

(1) the condition for which nontrivial solutions for the value σ_i exist in the system

(2) the condition that these σ_i values are such that the motion be stable, i.e., that the real parts of all the σ_i values be less than zero.

In order to fulfill condition (1) above, we substitute expressions (48) into equations (43) through (47) and realizing that the principle of superposition of solutions holds for linear differential equations, drop the summation notation and obtain the following relations:

1. Barge

   a) Lateral Force

$$\left\{\left[m_1'\ell' - Y'_{\dot\psi}\right]\sigma_i^2 + \frac{T'}{\lambda'}\left[\ell' x_p' + \lambda'\right]\sigma_i + \frac{T'}{\lambda'}\right\}\psi_i - \left\{m_2'\ell'\sigma_i^2 + Y'_\beta\sigma_i + \frac{T'}{\lambda'}\right\}\beta_i - \left\{\frac{\bar{x}_p\bar{\ell}' T'}{\lambda'}\sigma_i + \frac{T'}{\lambda'}\right\}\bar{\psi}_i + \frac{T'}{\lambda'}\bar{\beta}_i = 0 \qquad (49)$$

   b) Yawing Moment

$$\left\{n'\ell'^2\sigma_i^3 - N'_{\dot\psi}\sigma_i^2 + \frac{x_p' T'}{\lambda'}(\ell' x_p' + \lambda')\sigma_i + \frac{x_p' T'}{\lambda'}\right\}\psi_i - \left\{N'_\beta\sigma_i + \frac{x_p' T'}{\lambda'}\right\}\beta_i - \left\{\frac{x_p'\bar{x}_p'\bar{\ell}' T'}{\lambda'}\sigma_i + \frac{x_p' T'}{\lambda'}\right\}\bar{\psi}_i + \frac{x_p' T'}{\lambda'}\bar{\beta}_i = 0\,. \qquad (50)$$

2. Towboat

   a) Lateral Force

$$\left\{\left[\bar{m}_1'\bar{\ell}' - \bar{Y}'_{\dot{\psi}}\right]\sigma_\iota^2 + \frac{T'}{\gamma'}\left[\bar{\ell}'\bar{x}_p' - \gamma'\right]\sigma_\iota + \frac{T'}{\gamma'}\right\}\bar{\psi}_\iota$$
$$- \left\{\bar{m}_2'\bar{\ell}'\sigma_\iota^2 + \bar{Y}'_{\bar{\beta}}\sigma_\iota + \frac{T'}{\gamma'}\right\}\bar{\beta}_\iota - \left\{\frac{x_p'\,\ell'T'}{\gamma'}\sigma_\iota + \frac{T'}{\gamma'}\right\}\psi_\iota$$
$$+ \frac{T'}{\gamma'}\beta_\iota - \bar{Y}'_{\bar{\delta}}\sigma_\iota\,\bar{\delta}_\iota = 0 \qquad (51)$$

b) Yawing Moment

$$\left\{\bar{n}'\bar{\ell}'^2\sigma_\iota^3 - \bar{N}'_{\dot{\psi}}\sigma_\iota^2 + \frac{\bar{x}_p'T'}{\gamma'}(\bar{\ell}'\bar{x}_p' - \gamma')\sigma_\iota + \frac{\bar{x}_p'T'}{\gamma'}\right\}\bar{\psi}_\iota$$
$$- \left\{\bar{N}'_{\bar{\beta}}\sigma_i + \frac{\bar{x}_p'\,T'}{\gamma'}\right\}\bar{\beta}_i - \left\{\frac{x_p'\,\bar{x}_p'\,\ell'T'}{\gamma'}\sigma_\iota + \frac{\bar{x}_p'\,T'}{\gamma'}\right\}\psi_\iota$$
$$+ \frac{\bar{x}_p'\,T'}{\gamma'}\beta_i - \bar{N}'_{\bar{\delta}}\sigma_\iota\,\bar{\delta}_i = 0 \qquad (52)$$

c) Rudder

$$\bar{\delta}_\iota = a_o\,\bar{\psi}_i \qquad (53)$$

We now have a system of linear homogeneous equations in ψ_ι, β_ι, $\bar{\psi}_\iota$, $\bar{\beta}_\iota$, and $\bar{\delta}_\iota$. The condition that a nontrivial solution exists among them is that the determinant of their coefficients be identically equal to zero. This means that

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{vmatrix} = 0 \qquad (54)$$

where:

$$a_{11} = \left\{\left[m_1'\,\ell' - Y'_{\dot{\psi}}\right]\sigma_i^2 + \frac{T'}{\gamma'}\left[\ell' x_p' + \gamma'\right]\sigma_\iota + \frac{T'}{\gamma'}\right\}$$
$$a_{12} = -\left\{m_2'\,\ell'\sigma_\iota^2 + Y'_\beta\sigma_\iota + \frac{T'}{\gamma'}\right\} \qquad (55)$$

$$a_{13} = -\left\{ \frac{\bar{x}'_p \bar{\ell}' T'}{l'} \sigma_\iota + \frac{T'}{l'} \right\}$$

$$a_{14} = \frac{T'}{l'} \tag{55}$$

$$a_{15} = 0$$

$$a_{21} = -\left\{ n' \ell'^{2} \sigma_\iota^{3} - N'_{\dot\psi} \sigma_\iota^{2} + \frac{x'_p T'}{l'} \left[\ell' x'_p + l' \right] \sigma_\iota + \frac{x'_p T'}{l'} \right\}$$

$$a_{22} = -\left\{ N'_\beta \sigma_\iota + \frac{x'_p T'}{l'} \right\}$$

$$a_{23} = -\left\{ \frac{x'_p \bar{x}'_p \bar{\ell}' T'}{l'} \sigma_\iota + \frac{x'_p T'}{l'} \right\} \tag{56}$$

$$a_{24} = \frac{x'_p T'}{l'}$$

$$a_{25} = 0$$

$$a_{31} = -\left\{ \frac{x'_p \ell' T'}{l'} \sigma_\iota + \frac{T'}{l'} \right\}$$

$$a_{32} = \frac{T'}{l'} \tag{57}$$

$$a_{33} = \left\{ \left[\bar{m}'_1 \bar{\ell}' - \bar{Y}'_{\dot\psi} \right] \sigma_i^{2} + \frac{T'}{l'} \left[\bar{\ell}' \bar{x}'_p - l' \right] \sigma_\iota + \frac{T'}{l'} \right\}$$

$$a_{34} = -\left\{ \bar{m}'_2 \bar{\ell}' \sigma_\iota^{2} + \bar{Y}'_{\bar\beta} \sigma_\iota + \frac{T'}{l'} \right\}$$

$$a_{35} = -\bar{Y}'_{\bar\delta} \sigma_\iota$$

$$a_{41} = -\left\{ \frac{x'_p \bar{x}'_p \bar{\ell}' T'}{l'} \sigma_\iota + \frac{\bar{x}'_p T'}{l'} \right\}$$

$$a_{42} = \frac{\bar{x}'_p T'}{l'} \tag{58}$$

$$a_{43} = \left\{ \bar{n}' \bar{\ell}'^{2} \sigma_\iota^{3} - \bar{N}'_{\dot\psi} \sigma_\iota^{2} + \frac{\bar{x}'_p T'}{l'} \left[\bar{\ell}' \bar{x}'_p - l' \right] \sigma_\iota + \frac{\bar{x}'_p T'}{l'} \right\}$$

$$a_{44} = -\left\{ \bar{N}'_{\bar\beta} \sigma_\iota + \frac{\bar{x}'_p T'}{l'} \right\}$$

$$a_{45} = -\bar{N}_{\bar\delta} \sigma_\iota$$

$$a_{51} = 0$$

$$a_{52} = 0 \qquad (59)$$

$$a_{53} = a_o$$

$$a_{54} = 0 \qquad a_{55} = -1$$

Expansion of this determinant leads to the following relation:

$$\sum_{j=0}^{8} C_j \sigma^{8-j} = 0 \qquad (60)$$

where:

$$C_o = (ah - \ell d)(AH - LD)$$

$$C_1 = (ah - \ell d)(AJ + BH - LF - MD) + (aj + bh - \ell f - md)(AH - LD)$$

$$\begin{aligned} C_2 = {} & (ah - \ell d)(BJ + EH + LG - MF - ND) \\ & + (aj + bh - \ell f - md)(AJ + BH - LF - MD) \\ & + (bj + eh - \ell g - mf - jg)(AH - LD) \end{aligned}$$

$$\begin{aligned} C_3 = {} & (ah - \ell d)(EJ + Mg - NF) \\ & + (aj + bh - \ell f - md)(BJ + EH + Lg - MF - ND) \\ & + (bj + eh - \ell g - mf - jg)(AJ + BH - LF - MD) \\ & + (ej - mg - jf)(AH - LD) \end{aligned} \qquad (61)$$

$$\begin{aligned} C_4 = {} & (ah - \ell d)\, Ng + (aj + bh - \ell f - md)(EJ + Mg - NF) \\ & + (bj + eh - \ell g - mf - jg)(BJ + EH + Lg - MF - ND) \\ & + (ej - mg - jf)(AJ + BH - LF - MD) \\ & - jg\,(AH - LD) + g^2 \ell L \end{aligned}$$

$$\begin{aligned} C_5 = {} & (aj + bh - \ell f - md)\, Ng \\ & + (bj + eh - \ell g - mf - jg)(EJ + Mg - NF) \\ & + (ej - mg - jf)(BJ + EH + Lg - MF - ND) \\ & - jg\,(AJ + BH - LF - MD) + (g^2 \ell M + g^2 mL) \end{aligned}$$

$$C_6 = (bj + eh - \ell g - mf - jg)\,Ng$$
$$+ (ej - mg - jf)(EJ + Mg - NF)$$
$$- jg\,(BJ + EH + Lg - MF - ND)$$
$$+ (g^2 \ell\, N + g^2 mM + g^2 j\, L) \qquad (61)$$
$$C_7 = (ej - mg - jf)\,Ng - jg\,(EJ + Mg - NF)$$
$$+ (g^2 m\, N + g^2 j\, M)$$
$$C_8 = (-jg^2 N + gjM)$$

where:

$$a = -m_2'\, x_p'\, \ell'^2$$
$$b = \left[(m_1' - m_2' - x_p'\, Y'_\beta)\ell' - Y'_{\dot\psi}\right]$$
$$e = \left[T' - Y'_\beta\right]$$
$$d = -m_2' \ell'$$
$$f = -Y'_\beta \qquad (62)$$
$$g = -\frac{T'}{\ell'}$$
$$h = m_2'\, x_p' \ell'$$
$$j = \left[x_p'\, Y'_\beta - N'_\beta\right]$$
$$\ell = \left[n'\ell'^2 + m_2'\, x_p'^2\, \ell'^2\right]$$
$$m = -\left[N'_{\dot\psi} + x_p'\, \left([m_1' - m_2' - (x_p'\, Y'_\beta - N'_\beta)] - Y'_{\dot\psi}\right)\right]$$

$$A = -\bar m_2'\, \bar x_p'\, \bar\ell'^2$$
$$B = \left[(\bar m_1' - \bar m_2' - \bar x_p'\, \bar Y'_{\bar\beta})\bar\ell' - \bar Y'_{\dot{\bar\psi}}\right]$$
$$E = -\left[T' + \bar Y'_{\bar\beta} + a_o\, \bar Y'_{\bar\delta}\right]$$
$$D = -\bar m_2' \bar\ell' \qquad (63)$$
$$F = -\bar Y'_\beta$$
$$H = \bar m_2'\, \bar x_p' \bar\ell'$$
$$J = \left[\bar x_p'\, \bar Y'_{\bar\beta} - \bar N'_{\bar\beta}\right]$$

$$L = \left[\bar{n}'\bar{\ell}'^{2} + \bar{m}'_{2} \; \bar{x}'^{\,2}_{p} \, \bar{\ell}'^{2}\right] \tag{63}$$

$$M = -\left[\bar{N}'_{\dot{\psi}} + \bar{x}'_{p} \; \left(\left[\bar{m}'_{1} - \bar{m}'_{2} - (\bar{x}'_{p} \; Y'_{\bar{\beta}} - \bar{N}'_{\bar{\beta}})\right]\bar{\ell}' - \bar{Y}'_{\dot{\psi}}\right)\right]$$

$$N = \left[(\bar{x}'_{p} \; Y'_{\bar{\beta}} - \bar{N}'_{\bar{\beta}}) + a_{o} \, (\bar{x}'_{p} \; \bar{Y}'_{\bar{\delta}} - \bar{N}'_{\bar{\delta}})\right]$$

To fulfill condition (2) on page (14), we need look to the theory of equations as developed for polynomials. In Ref. 3, we find that the conditions for stability (the real parts of all the roots are less than zero) are stipulated by the following considerations:

(1) all $C_j > 0$ (64)

(2) all determintal relations obtained from

$$\begin{vmatrix} C_1 & C_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_3 & C_2 & C_1 & C_0 & 0 & 0 & 0 & 0 \\ C_5 & C_4 & C_3 & C_2 & C_1 & C_0 & 0 & 0 \\ C_7 & C_6 & C_5 & C_4 & C_3 & C_2 & C_1 & C_0 \\ 0 & C_8 & C_7 & C_6 & C_5 & C_4 & C_3 & C_2 \\ 0 & 0 & 0 & C_8 & C_7 & C_6 & C_5 & C_4 \\ 0 & 0 & 0 & 0 & 0 & C_8 & C_7 & C_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & C_8 \end{vmatrix} > 0 \tag{65}$$

By striking out:

first the first two rows and columns

i.e. $$\begin{vmatrix} C_1 & C_0 \\ C_3 & C_2 \end{vmatrix} > 0 \tag{66}$$

second the first three rows and columns

i.e. $$\begin{vmatrix} C_1 & C_0 & 0 \\ C_3 & C_2 & C_1 \\ C_5 & C_4 & C_3 \end{vmatrix} > 0 \tag{67}$$

and so on until all such expressions have been written including (65).

These inequalities, when satisfied, indicate stability for the system under consideration. For any two particular bodies, a towline length and towpoint locations on both the barge and towboat may be determined to give desirable stability characteristics. This, however, requires considerable numerical analysis as well as time, unless suitable computing facilities are available.

The development of the theory as presented here is considered to be a basis for future numerical investigation of the directional stability of any two bodies separated by a towline of arbitrary length attached to arbitrary points along the centerline of each body.

A SIMPLIFIED THEORY

The foregoing analysis has been devoted to the directional stability of a towed configuration of two bodies, separated by a towline of arbitrary length, attached at arbitrary points along the centerline of each of the bodies, and further confined by a control rule for the towing body proportional to its heading deviation.

It is evident from this analysis that the results obtained defy, without considerable numerical calculation, rational qualitative results. Thus in order to correlate qualitative as well as some quantitative predictions with data from test measurements, certain rather broad simplifications will be made.

It will be assumed that

(1) the towline is horizontal, and taut (as before);

(2) the barge does not roll or pitch

(3) the towboat proceeds on a straight and prescribed course regardless of the motion of the barge.

With this latter assumption, all the towboat functions vanish and only the barge functions remain.

It is also convenient to define the transformation and dimensionless coefficients presented on pages 9 and 10 as:

$$\begin{aligned} s &= \frac{Vt}{\lambda} \\ \lambda' &= \frac{\lambda}{\ell} \end{aligned} \qquad (68)$$

$$\dot{\psi} = \dot{\psi}'\left[\frac{V}{\ell}\right] , \quad \psi = \dot{\psi}'\left[\frac{V}{\ell}\right]^2 , \quad \dot{\beta} = \dot{\beta}'\left[\frac{V}{\ell}\right]$$

$$m_1' = \frac{M_1}{\frac{\rho}{2}\ell^3} , \quad m_2' = \frac{M_2}{\frac{\rho}{2}\ell^3} , \quad n' = \frac{I}{\frac{\rho}{2}\ell^5} \tag{68}$$

$$Y'_\beta = \frac{Y_\beta}{\frac{\rho}{2}\ell^2V^2} , \quad N'_\beta = \frac{N_\beta}{\frac{\rho}{2}\ell^3V^2} , \quad T' = \frac{T}{\frac{\rho}{2}\ell^2V^2}$$

$$Y'_{\dot\psi} = \frac{Y_{\dot\psi}}{\frac{\rho}{2}\ell^3V} , \quad N'_{\dot\psi} = \frac{N_{\dot\psi}}{\frac{\rho}{2}\ell^4V}$$

With these in mind, equations (43) and (44) reduce to:

Lateral Force

$$\left[m_1' - Y'_{\dot\psi}\right]\ddot{\psi}' + T'\left[1 + \frac{x_p'}{\ell'}\right]\dot{\psi}' + \frac{T'}{\ell'}\psi$$
$$- m_2' \ddot{\beta}' - Y'_\beta\dot{\beta}' - \frac{T'}{\ell'}\beta = 0 \tag{69}$$

Yawing Moment

$$n'\dddot{\psi}' - N'_{\dot\psi}\ddot{\psi}' + x_p' T'\left[1 + \frac{x_p'}{\ell'}\right]\dot{\psi}'$$
$$+ \frac{x_p' T'}{\ell'}\psi - N'_\beta\ddot{\beta}' - \frac{x_p' T'}{\ell'}\beta = 0 \tag{70}$$

Reasoning similar to that presented on pages 13 through 17 leads to the stability equation

$$\sum_{j=0}^{4} C_j{}^{4-j} = 0 \tag{71}$$

where

$$C_0 = n' m_2'$$

$$C_1 = n' Y'_\beta - m_2' N'_{\dot\psi}$$

$$C_2 = (Y'_{\dot\psi} - m_1')N'_\beta + x_p' T' m_2' \left(1 + \frac{x_p'}{\ell'}\right) - Y'_\beta N'_{\dot\psi} + n'\frac{T'}{\ell'}$$

$$C_3 = (x_p' Y'_\beta - N'_\beta)\left(1 + \frac{x_p'}{\ell'}\right)T' + \left[x_p'(Y'_{\dot\psi} - m_1' + m_2') - N'_{\dot\psi}\right]\frac{T'}{\ell'}$$

$$C_4 = (x_p' Y'_\beta - N'_\beta)\frac{T'}{\ell'} \tag{72}$$

The motion will be stable if and only if all the C_j values are positive and the following relations are satisfied:

$$\Delta_1 = \begin{vmatrix} C_1 & C_0 \\ C_3 & C_2 \end{vmatrix} = C_1 C_2 - C_0 C_3 > 0 \tag{73}$$

and

$$\Delta_2 = \begin{vmatrix} C_1 & C_0 & 0 & 0 \\ C_3 & C_2 & C_1 & C_0 \\ 0 & C_4 & C_3 & C_2 \\ 0 & 0 & 0 & C_4 \end{vmatrix} \approx (C_1 C_2 C_3 - C_1{}^2 C_4 - C_0 C_3{}^2) > 0 \tag{74}$$

In the case of the untowed body, it is noted (Ref. 1) that

$$\begin{aligned} N'_{\dot{\psi}} Y'_{\beta} + (m_1' - Y'_{\dot{\psi}}) N'_{\beta} &= -n' m_2' \bar{\sigma}_1 \bar{\sigma}_2 \\ n' Y'_{\beta} - m_2' N'_{\dot{\psi}} &= -n' m_2' (\bar{\sigma}_1 + \bar{\sigma}_2) \end{aligned} \tag{75}$$

where $\bar{\sigma}_1$ and $\bar{\sigma}_2$ are the dynamic stability indices of the untowed body. Then the C_j values may be written as

$$\begin{aligned} C_0 &= n' m_2' \\ C_1 &= -n' m_2' (\bar{\sigma}_1 + \bar{\sigma}_2) \\ C_2 &= \left[x_p' T' m_2' \left(1 + \frac{x_p'}{\ell'}\right) + n' \frac{T'}{\ell'} + n' m_2' \bar{\sigma}_1 \bar{\sigma}_2 \right] \\ C_3 &= \left[x_p' Y'_{\beta} - N'_{\beta} \right] \left(1 + \frac{x_p'}{\ell'}\right) T' + \left[x_p' (Y'_{\dot{\psi}} - m_1' + m_2') - N'_{\dot{\psi}} \right] \frac{T'}{\ell'} \\ C_4 &= \left[x_p' Y'_{\beta} - N'_{\beta} \right] \frac{T'}{\ell'} \end{aligned} \tag{76}$$

Without further mathematical manipulation, the following analysis may be made: by definition, all the C_j values must be positive. Then:

1) $C_0 > 0$ always

2) $C_1 > 0$ for:

   a) all dynamically stable untowed bodies, since

      σ_1 and $\sigma_2 < 0$;

   b) all dynamically neutrally stable untowed bodies, since

      $\bar{\sigma}_1 = 0$ and $\bar{\sigma}_2 < 0$;

   c) all dynamically unstable untowed bodies for which

      $|\bar{\sigma}_1| < |\bar{\sigma}_2|$, where $\bar{\sigma}_1 > 0$ and $\bar{\sigma}_2 < 0$.

   As a consequences of this, a towed body cannot be stable on course if $|\bar{\sigma}_1| > |\bar{\sigma}_2|$ and $|\bar{\sigma}_1| > 0$.

3) $C_2 > 0$ for:

   a) all dynamically stable untowed bodies, since the towing functions are positive and since both $\bar{\sigma}_1$ and $\bar{\sigma}_2$ are negative;

   b) all dynamically neutrally stable untowed bodies, since

      $\bar{\sigma}_1 = 0$ and $\bar{\sigma}_2 < 0$;

   c) all dynamically unstable bodies for which

      $$x_p' \, T' m_2' \left(1 + \frac{x_p'}{\ell'}\right) + n' \frac{T'}{\ell'} + n' m_2' \, \bar{\sigma}_1 \bar{\sigma}_2 > 0 \, .$$

4) $C_3 > 0$ for all towed bodies (see item 5 below), since by definition, $(x_p' \, Y'_\beta - N_\beta) > 0$, and usually $(Y'_{\dot\psi} - m_1' + m_2') > 0$, $N'_{\dot\psi} > 0$ and T' and $\ell' > 0$.

5) $C_4 > 0$ by definition. Since

$$(x_p' \; Y'_\beta - N'_\beta) = x_p' \left[\frac{\partial L'}{\partial \beta} - D' \right] - N'_\beta > 0 ,$$

this condition may be fulfilled either by increasing x_p' (moving the towpoint farther forward), or, if this is in a practical sense impossible, by increasing the drag in such a way that the static moment rate will remain unchanged or is decreased. The condition $C_4 > 0$ defines the location of the towpoint: it must always be a distance x'_p forward of the center of gravity of the ship, which is greater than the distance between the center of gravity and the center of pressure of the static force $Y'_\beta \beta$.

Assuming that the C_j values are all positive, it is necessary to examine the determinantal relationships

$$\text{a)} \quad \Delta_1 = - n' m_2' \;(\bar\sigma_1 + \bar\sigma_2) \left[x_p' \, T' \,(1 + \frac{x_p'}{\ell'}) \, m_2' \; + n' \frac{T'}{\ell'} + m_2' \, n' \; \bar\sigma_1 \, \bar\sigma_2 \right]$$

$$- n' m_2' \left\{ \left[x_p' \, Y'_\beta - N'_\beta \right] (1 + \frac{x_p'}{\ell'}) \, T' + \left[x_p' \; (Y'_{\dot\psi} - m_1' \; + m_2' \;) \right. \right.$$

$$\left. \left. N'_{\dot\psi} \right] \frac{T'}{\ell'} \right\} > 0$$

$$\sim - n' m_2' \;(\bar\sigma_1 + \bar\sigma_2) \; \bar\sigma_1 \; \bar\sigma_2 - T' \left\{ (\bar\sigma_1 + \bar\sigma_2) \left[x_p' \, m_2' \;\; (1 + \frac{x_p'}{\ell'}) + \frac{n'}{\ell'} \right] \right.$$

$$\left. + \left[x_p' \, Y'_\beta - N'_\beta \right] \left[1 + \frac{x_p'}{\ell'} \right] + \left[x_p' \;\; (Y'_{\dot\psi} - m_1' + m_2' \;) - N'_{\dot\psi} \right] \frac{1}{\ell'} \right\} > 0$$

$$\sim - n' m_2' \;(\bar\sigma_1 + \bar\sigma_2) \; \bar\sigma_1 \; \bar\sigma_2 \; - T' \left[(\bar\sigma_1 + \bar\sigma_2) \; Q + R \right] \; > 0 \qquad (77)$$

where

$$Q = x_p' \;\; m_2' \;\; (1 + \frac{x_p'}{\ell'}) + \frac{n'}{\ell'}$$

and

$$R = \left[x_p' \; Y_\beta' - N_\beta'\right]\left[1 + \frac{x_p'}{\ell'}\right] + \left[x_p' \,(Y_{\dot{\psi}}' - m_1' + m_2') - N_{\dot{\psi}}'\right]\frac{1}{\ell'}$$

and Q, R, and $n' \, m_2'$ must be positive. The following cases will now be examined:

i) Untowed body, dynamically stable. Here,

$$-n' m_2' \;(\bar{\sigma}_1 + \bar{\sigma}_2)\,\bar{\sigma}_1\,\bar{\sigma}_2 > 0$$

always for $\bar{\sigma}_1 \, \bar{\sigma}_2 > 0$ and $(\bar{\sigma}_1 + \bar{\sigma}_2) < 0$. Thus, the sense of the inequality depends on the magnitude of $T'R$; hence, to achieve stability, it is desirable to keep this term small. This may be accomplished by proper selection of x_p', the towpoint, and ℓ', the towline length.

ii) Untowed body, neutrally stable. The first term vanishes since $\bar{\sigma}_1 \, \bar{\sigma}_2 = 0$; thus R must be made small in order to satisfy the inequality, and insure stability. To accomplish this, the same reasoning is applied as in a).

iii) Untowed body, dynamically unstable. Here, $\bar{\sigma}_1 > 0$ and as long as $\bar{\sigma}_2 < 0$ and $|\bar{\sigma}_1| < |\bar{\sigma}_2|$ then

$$-n' m_2' \;(\bar{\sigma}_1 + \bar{\sigma}_2)\,\bar{\sigma}_1\,\bar{\sigma}_2 < 0$$

Thus, for stability, it is desirable to make R as small as possible and to increase the drag in such a fashion that the inequality is satisfied.

$$\text{b)}\quad \Delta_2 \sim C_1 C_2 C_3 - C_1^{2} C_4 - C_0 C_3^{2} > 0$$

$$\sim C_3 (C_1 C_2 - C_0 C_3) - C_1^{2} C_4 > 0 \qquad (78)$$

$$\sim C_3 \left[\Delta_1 - \frac{C_1^{2} C_4}{C_3} \right] > 0$$

If it is assumed that the conditions specified for Δ_1 have been met, then the remaining factor to consider is the magnitude of C_1^{2} (C_4/C_3). C_1 is determined by the values of $\bar{\sigma}_1$, $\bar{\sigma}_2$ and $n' m_2'$, and $0 << C_4/C_3 << 1$. It is therefore desirable to make C_4 as small as possible, still keeping it positive, and thus insure the stability of the system.

The criteria of directional stability will now be determined for the case where the towline has increased in length to a point where the terms containing its reciprocal are negligible. Mathematically, this is achieved by letting the towline length increase without limit. In this case, it is found that

$$\Delta_2 = -n' m_2' \; T' \left[x_p' Y'_\beta - N'_\beta \right] \left\{ m_2' \; (\bar{\sigma}_1 + \bar{\sigma}_2)(x_p' T' + n' \bar{\sigma}_1 \bar{\sigma}_2) + \left[x_p' Y'_\beta - N'_\beta \right] T' \right\} > 0$$

Since n', m_2', T' and $(x_p' \; Y'_\beta - N'_\beta)$ are positive, this expression will be positive if

$$\left\{ m_2' \; (\bar{\sigma}_1 + \bar{\sigma}_2)(x_p' T' + n' \bar{\sigma}_1 \bar{\sigma}_2) + (x_p' Y'_\beta - N'_\beta) T' < \right\} 0$$

or

$$(79)$$

$$(x_p' \; Y'_\beta - N'_\beta) T' < -m_2' \; (\bar{\sigma}_1 + \bar{\sigma}_2) \; (x_p' \; T' + n' \bar{\sigma}_1 \bar{\sigma}_2).$$

For all dynamically stable ships, the right-hand side of the inequality is positive; thus $(x_p' \, Y'_\beta - N'_\beta)T'$ must be kept as small as possible if the resulting motion is to be directionally stable. For dynamically unstable ships, the directional stability depends on $(x_p' \, T' + n' \, \bar{\sigma}_1 \, \bar{\sigma}_2) > 0$. Since in this case $\bar{\sigma}_1 \, \bar{\sigma}_2 < 0$, it must be specified that for directional stability, $x_p' \, T' > - n' \bar{\sigma}_1 \, \bar{\sigma}_2$ in addition to the condition that

$$(x_p' \, Y'_\beta - N'_\beta) \, T' < - m_2' \, (\bar{\sigma}_1 + \bar{\sigma}_2) \, (x_p' \, T' + n' \bar{\sigma}_1 \, \bar{\sigma}_2)$$

To investigate the consequences of decreasing the towline length to zero, i.e., either towing the ship from its bow or with a bridle, we discard the terms containing the towline length ℓ' in equations (69) and (70) on page 22 since they have no significance in this situation. The resulting stability equation is then a cubic of the form of

$$\sum_{j=0}^{3-j} \bar{C}_j \, \sigma_i = 0 \qquad (80)$$

where

$$\bar{C}_0 = n' \, m_2'$$

$$\bar{C}_1 = (n' Y'_\beta - m_2' \, N'_{\dot{\psi}}) = - n' \, m_2' \, (\bar{\sigma}_1 + \bar{\sigma}_2)$$

$$\bar{C}_2 = (Y'_{\dot{\psi}} - m_1') \, N'_\beta - Y'_\beta N'_{\dot{\psi}} + m_2' \, x_p' \, T' = n' m_2' \, \bar{\sigma}_1 \, \bar{\sigma}_2 + m_2' \, x_p' \, T'$$

$$\bar{C}_3 = (x_p' \, Y'_\beta - N'_\beta) \, T' \qquad (81)$$

The criteria for stability specifies that all the $\bar{C}_j$ values must be positive and that the following determinant is satisfied:

$$\bar{\Delta} = \begin{vmatrix} \bar{C}_1 & C_0 & 0 \\ \bar{C}_3 & \bar{C}_2 & \bar{C}_1 \\ 0 & 0 & \bar{C}_3 \end{vmatrix} > 0 \qquad (82)$$

This reduces to $(\bar{C}_1 \bar{C}_2 - \bar{C}_0 \bar{C}_3) > 0$ or

$$-n'm_2'^{\,2} (\bar{\sigma}_1 + \bar{\sigma}_2)\left[n' \bar{\sigma}_1 \bar{\sigma}_2 + x_p' T'\right] - n'm_2' (x_p' Y'_\beta - N'_\beta) T' > 0$$

Hence, the motion will be stable if

$$-m_2' (\bar{\sigma}_1 + \bar{\sigma}_2)\left[n' \bar{\sigma}_1 \bar{\sigma}_2 + x_p' T'\right] > (x_p' Y'_\beta - N'_\beta) T' \qquad (83)$$

For dynamically stable ships, the left-hand side of the inequality is positive; consequently, the stability or instability of the towed ships is essentially a matter of towpoint location.

For dynamically unstable ships, however, the extent of instability will be an important factor in determining the directional stability, since the sign of the left-hand side of the inequality will depend on the sign of $(n'\bar{\sigma}_1 \bar{\sigma}_2 + x_p' T')$. Thus, for the resulting motion to be stable, $(n'\bar{\sigma}_1 \bar{\sigma}_2 + x_p' T') > 0$. Hence, the dynamic stability criterion $\bar{\sigma}_1$ must be small if this condition is to be satisfied.

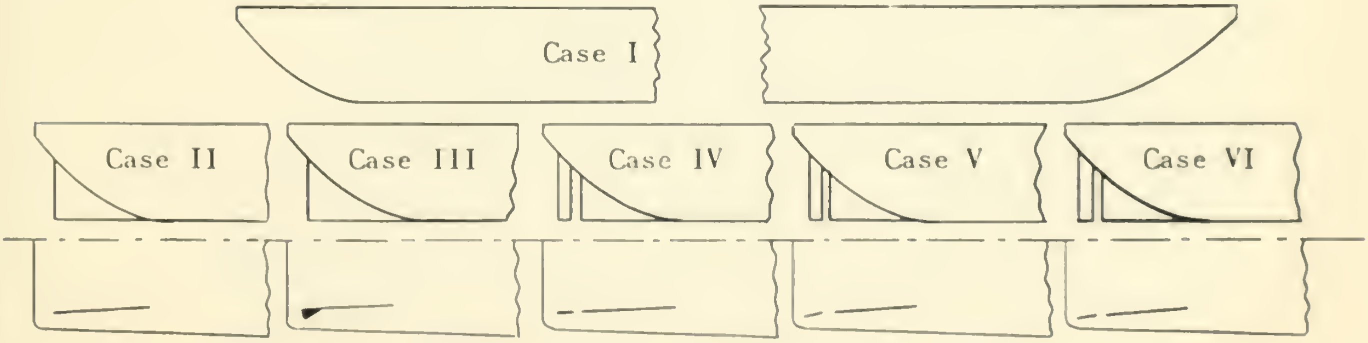
Reducing towline length, however, is of very little practical consequence, since an excessive decrease in towline length causes hydrodynamic interaction between the towing and towed ships, a fact that will cause variation in the hydrodynamic "constants". Thus, the basic assumptions of the analysis break down and the conclusions derived become untrustworthy.

NUMERICAL ANALYSIS

The results of an investigation to determine the hydrodynamic coefficients of a typical barge form with various skeg configurations made on the rotating arm of the Experimental Towing Tank afford the data necessary for an analysis based on the foregoing treatment.

The particular model considered had been found to be stable according to the free-towing standards set by the Experimental Towing Tank. These standards stipulate that if a barge is towed from its bow with a towline length of twice the barge length, that the barge should not be laterally displaced more than $\pm\frac{1}{2}$ of its beam. The towing device is a carriage that proceeds along the tank rail at constant speed for the major portion of its travel.

The accompanying sketch shows the bare hull of the barge model together with the skeg modifications investigated on the rotating arm (Ref. 6). The accompanying tabulation gives details of the configurations as well as the comparable results of the standard free-towing stability criterion set by the Experimental Towing Tank where these results are applicable.



Model Particulars:

$n' = .00219$ $m_2' = 0.0351$ $m_1' = 0.0219$

$\ell = 6.5$ ft. $x_p' = 0.508$ $l' = 2.0$

$\rho = 1.94$

Case I - bare hull (unsatisfactory by free-towing method)

Case II - bare hull and skegs (5° to ℄) (unsatisfactory by free-towing method)

Case III - bare hull, skegs and stabilizing prisms (satisfactory by free-towing method)

Case IV - bare hull and slotted skegs (not tested by free-towing method)

Case V - bare hull and slotted skegs deflected to 6° (not tested by free-towing method)

Case VI - bare hull and slotted skegs deflected to 12° (not tested by free-towing method)

In each case the model was tested on the rotating arm at a speed length ratio of $\frac{V}{\sqrt{\ell}} = 0.463$ where V is measured in knots. The following values of the hydrodynamic coefficients were determined:

Case	Y'_β	N'_β	$D' = T'$	$Y'_{\dot\psi}$	$N'_{\dot\psi}$
I	0.008	0.010	-0.00250	0.0038	-0.0020
II	0.020	0.0057	-0.00280	0.0049	-0.0029
III	0.020	0.0049	-0.00324	0.0055	-0.0034
IV	0.022	0.0069	-0.00280	0 0038	-0.0028
V	0.020	0.0060	-0.00280	0.0037	-0.0029
VI	0.019	0.0058	-0.00280	0.0029	-0.0030

Calculation in accordance with the modified development (pg.21) and Ref. 1 leads to the following values for the untowed stability criteria ($\bar{\sigma}_1$) together with the towed stability

criteria σ_i as well as C_j and Δ_i .

	CASE I	CASE II	CASE III	CASE IV	CASE V	CASE VI
$\bar{\sigma}_1$	+1.047	+0.237	+0.0735	+0.363	+0.303	+0.311
$\bar{\sigma}_2$	−2.188	−2.131	−2.195	−2.268	−2.197	−2.222
$C_0 \times 10^4$	0.7687	0.7687	0.7687	0.7687	0.7687	0.7687
$C_1 \times 10^4$	0.8772	1.4559	1.6314	1.4646	1.4559	1.4691
$C_2 \times 10^4$	−1.06	1.270	0.630	0.0239	0.1448	0.1248
$C_3 \times 10^4$	−0.0532	0.3259	0.4227	0.3112	0.3082	0.2934
$C_4 \times 10^4$	−0.0742	0.0624	0.0852	0.0602	0.0588	0.0546
$\Delta_1 \times 10^8$		0.1426	0.7029	−0.2042	−0.0261	−0.0422
$\Delta_2 \times 10^{12}$		−0.0858	0.0704	−0.1926	−0.1326	−0.1302
σ_1	0.82	0.576	−0.0311	0.1254	0.0905	0.0919
σ_2	0.044	±0.4775i	±0.509i	±0.4606i	±0.4634i	±0.4507i
σ_3	±0.248i	−0.1935	−0.2405	−0.1735	−0.1800	−0.1744
σ_4	−1.873	−1.8155	−1.8195	−1.9825	−1.895	−1.9200

The tabulation shows that none of the cases considered is dynamically stable since in each case one of the stability roots ($\bar{\sigma}_i$) is positive. Of all the skeg configurations, only Case III produced a directionally stable barge when towed (since this is the only condition where all of the real parts of the values are negative). Case III was also considered stable as a result of the free-towing tests previously described. However, the drag of the barge in this case is noticeably higher than for any of the others.

It will be seen that none of the other skeg configurations creates such high resistance. While these other cases are not directionally stable, it appears that the instability may well be reduced without creating excessive drag. It does not seem too unreasonable to assume that further experimental

investigation will yield skeg forms that will produce directionally stable barges without adverse drag increments.

The presences of complex σ_i values (paired in the table) indicates that the motion is characterized by oscillatory modes. In all cases except Case III, the oscillation is undamped and tends to build up with time. This fact imposes continuous alternate yawing on the towed ship. In Case III, which is directionally stable, the oscillations still appear but are damped and tend to diminish with time.

CONCLUSIONS

The preceding analysis shows that while the stability criteria and equation, for the case when two bodies are considered, have been determined, their evaluation for a given set of physical and hydrodynamic constants is extremely laborious. It is difficult, if not impossible. to gain an extensive amount of qualitative information from the coefficients of the stability equation and the stability criteria because of the complicated way in which the various terms comprising these expressions appear.

In the simplified analysis where only the towed body is considered the following generalizations may be made.

1. If a towed ship is to be directionally stable, the towpoint must be located forward of its center of gravity greater than the distance between the center of gravity and the center of pressure of the static lateral force.

2. Depending upon its degree of stability, mass, towpoint, resistance, and moment of inertia, a dynamically stable ship may become unstable when towed with a long enough towline.

3. Depending upon its towpoint, mass, moment of inertia, and damping functions, a dynamically unstable ship may become directionally stable when towed with a short enough towline, providing that there is negligible hydrodynamic interaction between it and the towing ship.

4. The motion of a towed body, which may be either di-

rectionally stable or unstable, is often characterized by oscillatory modes.

5. In cases where a ship is extremely unstable dynamically, no possible towing configuration can make it directionally stable.

6. Increase in hull drag aft causes variation in each of the hydrodynamic parameters. Sufficient drag will cause most towed ships to become directionally stable. The experimental data presented indicate that the drag increments may be minimized and directional stability achieved by alteration and improvement of skeg configurations.

APPENDIX [illegible]

NOMENCLATURE

All linear dimensions are in feet and angular dimensions in radians. The time unit is the second.

ℓ	barge length
x_p	barge towline point of attachment forward of barge center of gravity
CG	barge center of gravity
ℓ	towboat length
x_p	towboat towline point of attachment aft of towboat center of gravity
$\overline{CG}$	towboat center of gravity
ψ	barge heading angle
β	barge yaw angle
$\bar{\psi}$	towboat heading angle
$\bar{\beta}$	towboat yaw angle
$\bar{\delta}$	towboat rudder angle
$\bar{\delta}^*$	towboat rudder angle order
t	time
$\bar{t}$	towboat rudder time lag
V	barge linear velocity
$\bar{V}$	towboat linear velocity
ℓ	towline length
L	over-all length of tow, i.e., from bow of towboat to stern of barge
M	mass of barge
$\bar{M}$	mass of towboat
M_1	longitudinal virtual mass of barge
M_2	lateral virtual mass of barge
$\bar{M}_1$	longitudinal virtual mass of towboat
$\bar{M}_2$	lateral virtual mass of towboat
I_B	moment of inertia of barge
$\bar{I}_B$	moment of inertia of towboat
I	virtual moment of inertia of barge

$\bar{I}$	virtual moment of inertia of towboat
I_w	moment of inertia of displaced fluid of barge
$\bar{I}_w$	moment of inertia of displaced fluid of towboat
k_1	longitudinal virtual mass coefficient of barge
k_2	lateral virtual mass coefficient of barge
$\bar{k}_1$	longitudinal virtual mass coefficient of towboat
$\bar{k}_2$	lateral virtual mass coefficient of towboat
k'	virtual inertia coefficient of barge
$\bar{k}'$	virtual inertia coefficient of towboat
T	towline tension
ρ	density of water
Y	hydrodynamic lateral force of barge
N	hydrodynamic lateral moment of barge
$\bar{Y}$	hydrodynamic lateral force of towboat
$\bar{N}$	hydrodynamic lateral moment of towboat
$\cdot$ $\cdot\cdot$ $\cdot\cdot\cdot$	first, second, and third derivatives with respect to time
$'$	indication of dimensionless coefficient
	subscript notation denotes a partial derivative with respect to the subscript; thus $\frac{\partial Y}{\partial \beta} = Y_\beta \qquad \frac{\partial \bar{Y}}{\partial \bar{\beta}} = \bar{Y}_{\bar{\beta}}$ etc.
m_1'	dimensionless longitudinal virtual mass coefficient of barge
m_2'	dimensionless lateral virtual mass coefficient of barge
$\bar{m}_1$	dimensionless longitudinal virtual mass coefficient of towboat
$\bar{m}_2$	dimensionless lateral virtual mass coefficient of towboat
n'	dimensionless virtual moment of inertia coefficient of barge
$\bar{n}'$	dimensionless virtual moment of inertia coefficient of towboat

All other auxiliary symbols are defined where they appear in the text. The exact expressions for the dimensionless factors are given in each instance where they are first introduced.

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